Closing today: 4.3 (max/min)
Closing Fri: 4.4 (L'Hopital's rule)
Closing Mon: 4.4-5 (graphing)
Closing next Wed: 4.7(applied max/min)
Final Exam, Saturday, March 11
1:30-4:20pm, Kane 130

4.4 L'Hopital's Rule

First, recall as we discussed many times at the beginning of the term: (Assuming f and g cont. at a)

$$\lim_{x \to a} \frac{f(x)}{g(x)} = ??$$

$$f g(a) \neq 0$$
, then done! Ans $= \frac{f(a)}{g(a)}$.

If g(a) = 0 and $f(a) \neq 0$, then examine each side of x=a (look at the sign) Ans $= \infty, -\infty$, or DNE.

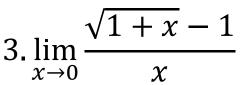
If g(a) = 0 and f(a) = 0then use algebra to rewrite in a form that 'cancels' the denominator.

2. $\lim \frac{\sin(x)}{x}$ L'Hopital's Rule (0/0 case) Suppose g(a) = 0 and f(a) = 0and f and g are differentiable at x = a, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Examples:

 $1.\lim_{x \to 4} \frac{16 - x^2}{4 - x}$



 $\chi \rightarrow 0$

Aside: Sketch of derivation Assume g(a) = 0 and f(a) = 0(These explanations are for the case when g'(a) is not zero).

Explanation 1 (def'n of derivative)

$$\frac{f'(a)}{g'(a)} = \frac{\lim_{x \to a} \frac{f(x) - f(a)}{x - a}}{\lim_{x \to a} \frac{g(x) - g(a)}{x - a}}$$

provided these limits exist we have:

$$\frac{f'(a)}{g'(a)} = \lim_{x \to a} \frac{\frac{f(x) - f(a)}{x - a}}{\frac{g(x) - g(a)}{x - a}}$$
$$= \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)} = \lim_{x \to a} \frac{f(x)}{g(x)}$$

Explanation 2 (tangent line approx.): The tangent lines for f(x) and g(x) at x = a are

$$y = f'(a)(x - a) + 0$$

 $y = g'(a)(x - a) + 0$

And we know these approximate the functions f(x) and g(x) better and better the closer x gets to a, so Thus,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(a)(x-a)}{g'(a)(x-a)} = \frac{f'(a)}{g'(a)}$$

Sometimes you have to use it more than once.

Example:

$$\lim_{x \to 1} \frac{x - \sin(x - 1) - 1}{(x - 1)^3}$$

L'Hopitals rule can also be used directly for the ∞/∞ case

 $3.\lim_{x\to\infty} xe^{-3x}$

Example: 1. $\lim_{x \to \infty} \frac{5x + 7}{6 + 13x}$

$$4.\lim_{x\to\infty}\frac{3x+1}{\sqrt{9+4x^2}}$$

$$2.\lim_{x\to\infty}\frac{\ln(x)}{x}$$

Other indeterminant forms: **0**· ∞ : (rewrite as a fraction) $\lim_{x \to 0^+} x \ln(x)$ ∞ - ∞: (combine into a fraction)
(example from our midterm) $\lim_{t \to \infty} \frac{2}{t(1+3t)^2} - \frac{2}{t}$

 $\lim_{x\to 0^+} x e^{1/x}$

$$0^{0}, \infty^{0}, 1^{\infty}$$
 : (Use In())

$$\lim_{x \to \infty} \left(1 + \frac{2}{x} \right)^x$$

 $\lim_{x\to 0^+} x^x$

Aside (you don't need to know this): You can use the techniques just

The formula for compound interest is discussed to find this limit and

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = starting balance, A = end balancer = annual rate, n = number of timesinterest is compounded each year,t = number of years

In some bank accounts interest is computed once a month, for some every day, for some every second. If you wanted interested to always be computed (continuously), then the new formula would be

$$A = \lim_{n \to \infty} P\left(1 + \frac{r}{n}\right)^{nt}$$

You can use the techniques just discussed to find this limit and determine it is

$$A = Pe^{rt}$$

This is the continuous compounding interest formula.